THE EFFECT OF WAVE DIRECTIONALITY ON EXTREME SIGNIFICANT WAVE HEIGHT PREDICTIONS

Summary

The extreme wave conditions in the open sea are a key criteria for the design of ships and offshore platforms. Design is typically based on high return period values for omni-directional significant wave heights. However, some situations require directional values of extreme waves but suitable approaches to deal with directional models that would preserve the consistency of the directional extremes with the omni-directional ones are still not widely available. In this paper, time series of hindcast significant wave heights from the northern Atlantic Ocean have been studied using the peaks over threshold (POT) approach for eight directional sectors (with $\Delta \theta = 45^\circ$) and fitted to generalized Pareto distribution (GPD). The distributional properties of the 100-year significant wave heights are estimated and the implications for design discussed.

Key words: significant wave height, wave direction, extremes prediction, Pareto distribution

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Sažetak

Ekstremni valni uvjeti na otvorenom moru su ključni kriterij za proces dimenzioniranja brodova i odobalnih (eng. „offshore”) platformi. Dimenzioniranje se tipično temelji na poznavanju značajne valne visine za velike povratne periode, neovisno o smjeru. Ipak, za pojedine slučajeve potrebno je uzeti u obzir direkcijske vrijednosti ekstremnih valova, iako prikladni načini modeliranja prognoze ekstremnih valova po direkcijskim smjerovima, a da se pritom očuva konzistencija kao u slučaju omni-direkcijskog modela nisu uvelike dostupni. U ovom radu, iz seta prognoziranih značajnih valnih visina s područja sjevernog Atlantskog oceanata formirani su uzorci valnih visina korištenjem metode prekoračenja praga (POT) za osam sektora smjera vala (sa $\Delta \theta = 45^\circ$), koji su opisani modelom generalizirane Pareto distribucije (GPD). Analizirana su svojstva dobivenog distribucijskog modela za 100-godišnje vrijednosti valnih visina te je diskutiran utjecaj istih na proces dimenzioniranja.

Ključne riječi: značajna valna visina, smjer vala, prognoza ekstrema, Pareto distribucija
1. Introduction

Environmental design criteria for ships and offshore platforms have inherent uncertainties, which are functions of climate variability in time and space, and storm peak direction and track. The quality of estimation of design criteria is further dependent on inherent data quality and sample size. In addition to the regular omni-directional criteria, the design of offshore structures can be further improved by taking into account the directional distribution of the environment and its extremes.

In the case of ships there is directionality in short term responses as results from the directional spreading of ocean waves, as well as long term variability, which can be felt for ships that have fixed routes, while other ships tend to have a more uniform distribution of meeting waves from a given direction [1]. For the short term responses use can be made of climatic spectra that are becoming available [2]. In fact the models for ships are much complicated by the fact that captains avoid bad weather by changing heading temporarily, which can introduce additional complications in an appropriate model of long term directional trends [3].

In most regions, but particularly hurricane-dominated regions (e.g. Gulf of Mexico), and in regions where extra tropical storms prevail (e.g. Northern North Sea), extremal properties of storms are dependent on the storm direction. Accordingly, sea state design criteria for the offshore facilities which are not symmetrical are frequently provided by the directional criteria in order to optimize the design of the structure for the directional environment. For example, frequently used return periods of 50 to 100 years of the significant wave heights can be specified for each of the eight 45° wide sectors in addition to the omni-directional case. However, debate on how to appropriately approach to the modelling of wave extremes with the wave directionality included is still open [4]. Besides the discussion about the most suitable method to describe the wave height extremes, there is still no consensus on how should be the directional model that would treat wave extremes in a consistent way in comparison to the corresponding omni-directional model. In other words it is necessary that the probability of exceedance of certain wave height from a given direction obtained with the directional model is equal as for the omni-directional model. Jonathan and Ewans argued that the risk-cost basis could be the most objective method for optimizing directional criteria, while preserving overall reliability [5].

In this study extreme analysis has been employed by fitting the generalized Pareto distribution (GPD) to samples obtained from a hindcast data set of significant wave heights from northern Atlantic Ocean that was produced within an European project [6]. From the wave data set, samples of the peaks over threshold (POT) for each of the eight directional sectors, with the 45° width, have been formed and modelled with GPD. Theory along with examples of the extreme value modelling can be found for example in Coles [7]. There are several authors that marked this branch of exploration. Coles and Walshaw modelled extremes of wind speeds as a function of their direction [8]. Robinson and Tawn applied a Fourier model to obtain smooth description of the extremal behaviour of the sea currents [9]. Ferreira and Guedes Soares [10] applied the POT to obtain extreme predictions of significant wave height and Guedes Soares and Scotto [11] compared the POT predictions with other methods. Coles and Tawn tested a Bayesian approach to form extreme value estimation [12].

What is common to these as well to the other articles that refer to estimation of directional extremes is that all reported higher estimates for 100-year maximum significant wave height from a directional model in comparison to the value obtained with model that ignore directionality. Similarly, this notion can be extended also to the extreme value models that treat other covariates, e.g. seasonality, in comparison to ones that ignore them, as reported for example in studies of Mendez et al. [13] and Mackay et al. [14]. Thus, along with the
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A discrete GPD directional model for estimation of the extreme significant wave heights, discussion about the suitable approaches and methods for directional modelling will be given here.

The paper is arranged as follows. In Section 2, the hindcast wave height data set will be presented along with the outline of the GPD model, used to characterize the extreme value behaviour of the data. In Section 3, parameters of the discrete GPD directional model along with the estimates of high quantiles are presented. In following Section 4, discussion about obtained results and inferences from them are given. To finish, some concluding remarks are given as well as references used for this study.

2. Data and methods

2.1. Wave height data set

This study has been performed on the data set obtained by hindcasting from the wind data sets, at three hourly intervals over the long period from January 1958 to December 2001. The quality of the hindcast results was assessed by comparing them with buoy data [15]. The present data set is from one of the grid points of the database, which is located at 9.5W; 45N in the northern Atlantic Ocean. Averaging data over time reduces the sampling variability thus the applied three hour interval is commonly used as a compromise between statistical stability and adequate sampling of changes in sea states.

This 44 years long wave heights series along with the corresponding wave directions can be considered as a sufficiently large data set to obtain a reliable directional model. Figure 1 indicates the frequency distribution of the data over the directional sectors. Directional sectors are defined clockwise from the North, meaning that the first sector covers waves going to the range of angles 0° to 45° and so forth.

![Fig. 1 Frequency distribution of mean wave direction over the directional sectors](image)

For modelling purposes, significant wave heights at the peaks of the storms are isolated together with the corresponding wave direction at the storm peak, henceforth referred to as storm peak direction. So each significant wave height at storm peak is separated according to the corresponding storm peak direction and placed into one of the eight directional bins with the angular width of 45°. In that manner POT samples for each of directional sectors are obtained which are used for extreme value modelling. Figure 2 illustrates the distribution of
the higher waves (over 3m) and storm peaks across the directional sectors. It is apparent that storms are frequent in sectors 3 and 4 and somewhat in sector 2, while the rest of the sectors are not abundant with storms, especially sector 8 which is practically without of the storms.

Fig. 2 Number of storm peaks and exceedances over threshold of 3m across directional sectors (for clarity peaks are multiplied by 10, exceedances are unchanged)

2.2. Generalized Pareto distribution

Extreme value modelling in this study has been obtained by fitting the generalized Pareto distribution (GPD) to peaks over threshold (POT) samples of significant wave heights, for two cases: (i) having directionality included, i.e. directional model or (ii) by neglecting directionality, i.e. omni-directional model.

Peaks over threshold (POT) method is the extension of the classical block maxima approach, where the key idea is to select the data not from the series of time blocks but from the upper segment of data set by using the threshold in order to extract exceedances or just peaks over a threshold. POT method is based on the fact that if block maxima have approximating generalized extreme value (GEV) distribution then excesses over high enough threshold u have the corresponding approximate distribution within the Generalized Pareto distributions (GPD) family, which has representation

\[ H(y) = 1 - \left( 1 + \frac{\xi y}{\sigma^*} \right)^{-1/\xi^*} \]  

(1)

There is consistency between the GEV and GPD model, meaning that the parameters can be related by \( \xi^* = \xi \) and \( \sigma^* = \sigma + \xi (\mu - \mu) \). In order to obtain return level first exceedances rate of threshold should be specified as \( \lambda = P(X > u) \), which is related to the GEV parameters by

\[ \lambda = 1 - \exp \left\{ -\frac{1}{N} \left[ 1 + \xi \left( \frac{u - \mu}{\sigma} \right) \right]^{-1/\xi^*} \right\} \]  

(2)

It follows that the mean crossing rate of any level \( (x > u) \) is

\[ \lambda \left( 1 + \xi (x - u)/\sigma^* \right)^{-1/\xi^*} \]  

(3)
By setting the Equation 1 equal to $1/m$ the $m$-year return level $q_m$ is obtained as

$$q_m = u + \frac{\sigma^*}{\xi^*} \left( (\lambda m)^{\xi^*} - 1 \right)$$

(4)

According to the extreme value theory (EVT), GPD model is an appropriate model for the distribution of significant wave heights excesses over a suitable chosen high threshold value. Actually, for the asymptotic arguments to hold within POT approach, the data in the sample should be independent, which is a plausible assumption.

Parameter estimation has been obtained by employing maximum likelihood methodology. In assessing the accuracy of maximum likelihood estimators appropriate profile likelihood functions were used as described in Coles [7].

3. Modelling significant wave height extremes

3.1. Modelling extremal properties incorporating directionality

A directional model has been produced by fitting the GPD to significant wave height POT samples for each of the eight directional sectors. In that manner, the GPD model has been used to describe the extremal behaviour of wave heights while taking into account wave directionality.

In order to give an insight to the distributional properties of the obtained discrete directional model the results of the GPD model for sector 3, which has the largest number of data, are presented on Figure 3.

![Diagnostic plots for discrete directional GPD model for sector 3 (threshold=10m)](image)

Fig. 3 Diagnostic plots for discrete directional GPD model for sector 3 (threshold=10m)

It is important to mention that sectors with less data have substantially larger variation of the model’s parameters. In that sense, the extreme analysis for sector 8, which has the smallest number of data, i.e. practically no storms at all, is useless. In the Figures 4-6 the
estimates of the 100-yr return values along with its confidence intervals, as well as parameters of the distributions and corresponding confidence intervals for directional sectors with minimum (sector 7), medium (sector 2) and maximum (sector 3) number of data are presented. From the presented graphs, the instability of the model’s parameters and corresponding high quantiles for sectors with less number of data can be easily noticed. Additionally, the gap between confidence envelopes and the parameter’s or return level’s estimates are therefore wider in sectors with less data.

Threshold selection is another issue that causes substantial variation of the extreme estimates in the case of directional modelling. Here for each of the directional sectors its own threshold value/range has been used. Point estimates of 100-yr return levels varied over directional sectors from lowest 6.5m in sector 7 to highest 17m in sector 3. Spreading of the extreme estimates over the examined threshold range is the largest in the sector 2 which can be related to the oscillation of the corresponding shape parameter around zero leading to potential overestimation of the extreme estimates.

Fig. 4. Variation of the 100-yr return levels on the left side and the GPD parameters on the right side across the examined threshold range are presented for SECTOR 7 (minimum data)

Fig. 5. Variation of the 100-yr return levels on the left side and the GPD parameters on the right side across the examined threshold range are presented for SECTOR 2 (medium data)
3.2. Estimation of omni-directional extremes

The omni-directional model neglects the directional variability of the observed storm peaks, so in this case the GPD model has been employed to all significant wave heights no matter of the directions at the storm peaks. To present the results of the omni-directional model diagnostic plots are given in Figure 7.

Fig. 7 Diagnostic plots for omni-directional GPD model (threshold=10m)

The variation of the distributional properties of the omni-directional GPD model and the corresponding 100 year return levels across the examined threshold range are presented in Figure 8.
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Fig. 8. Variation of the 100-yr return levels on the left side and the GPD parameters on the right side across the examined threshold range are presented for omni-directional model.

Diagnostic plots indicate good fit of the omni-directional GPD model to the data. The estimate of the 100-yr return level is around 17m which can be compared with the results obtained with directional model in sectors 2 and 3 while the other sectors produce significantly smaller values as can be expected due to smaller number of the storms.

4. Discussion

To summarize the results obtained with the directional and omni-directional GPD model, the parameters of the distributions are presented in Figure 9, while in Figure 10 estimates of the 100-yr return levels are shown.

Fig. 9. Variation of the scale and shape parameters across the range of examined thresholds for: directional model (dots) and omni-directional model (lines)
From these figures the pattern of the directional model over the sectors can be identified. For sectors with the most severe storms (sectors 2 and 3) the directional model produces similar or even overestimates the values of the 100-yr significant wave heights in comparison to the values obtained with the omni-directional model. In other sectors with less severe storms, estimates of 100-yr significant wave heights produced with directional model are smaller than the values obtained with omni-directional model.

Similar conclusions were reported by other authors in their studies [5,16]. In general, most of the studies reported higher return values from non-stationary models, i.e. models that incorporated the covariate effects, e.g. directionality or seasonality. Such a statement could be valid since it is plausible to have higher estimates of extreme levels of wave heights, in comparison to the omni-directional model, in sectors that contain more extremes than other sectors. Reasoning for this is that the extreme wave heights in severe sectors belong to different parent population than smaller wave heights that are numerous in the sample of all waves and especially in samples of other sectors with not many severe storms, leading to expected underestimation of extremes obtained with omni-directional model.

In order to give a smoother representation of the directional model, Fourier expansion can be used to describe the variability of the parameters of the directional model over the sectors under the assumption that the parameters vary slowly [5]. Regardless of the type of directional model, discrete or continuous, it can be used in engineering applications to improve the offshore structural design. When designing offshore structures only according to the results of a directional model one should pay attention not to violate the necessary preservation of the overall reliability, by matching the overall probability of non exceedance from all of the direction bands equal to the omni-directional probability. There are several different approaches how to form directional design criteria in consistent way, where family of so called “risk-cost” criteria [5] is the most plausible one in authors opinion. However there is still debate which of the approaches is the most suitable, so hopefully this paper will stimulate discussion.

5. Conclusion

It is important to incorporate the inherent directionality of the sea into the extreme value model when designing non symmetrical offshore structures. Consistency in design can be
achieved by taking into account different wave loads coming from different directions. In that sense the discrete directional GPD model of wave heights has been obtained for a data set from the North Atlantic Ocean. The data set has been divided into the eight directional bins with the angular width of 45°. From each directional sector peak over threshold (POT) samples of significant wave heights have been obtained by selecting only the wave heights at the peaks of the storms with the corresponding wave directions. For each of the eight directional sectors estimates of the 100 year return levels of significant wave heights have been generated. The obtained values vary from 6.5m in sector with the smallest number of data and probably very few observed storms to the values around 17m which were obtained in sector 3 which has the maximum number of data and supposedly the most severe sea conditions.

In order to evaluate the results derived with the directional model, the direction-independent GDP model has been obtained on the POT sample as well. The estimate of the 100 year significant wave height obtained with the omni-directional model is similar but slightly lower than the one obtained with the directional model for the most severe sector. Plausibility of the obtained results confirms the studies from other authors [5,16], which all supported the conclusion that omni-directional model underestimate the directional model in the most severe sector. In order to use the results of the directional model in engineering applications in conservative fashion one have to bear in mind that overall probability of non exceedance with directional model should not be less than corresponding probability of the omni-directional model. The optimal procedure for forming the directional design criteria is still open question. Hopefully this paper will further provoke some constructive discussion about this issue.

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6. References

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