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ON THE DISPLACEMENTS OF CONTAINER SHIP HULL GIRDER UNDER TORSION

Summary

An analytical approach to torsion of thin-walled beams of open section with one plane of symmetry is considered. The theory of torsion of thin-walled beams of open section with influence of shear, based on the classical Vlasov's theory of thin-walled beams of open section, as well as the Umansky's theory for closed-open sections, is applied. The general transverse loads act in the beam walls, reduced to the moments of torsion with respect to the principal pole (torsion/shear centre) only. The beam will be subjected to torsion with influence of shear with respect to the principal pole and in addition to bending due to shear in the horizontal plane trough the principal pole. The obtained analytical expressions for displacements are applied in the analysis of displacements of the modern container ship hull girder subjected to torsion, as well as in the parametric analysis of simple U sections. Comparisons with the finite element method by applying shell elements are provided.

Key words: *Theory of thin-walled beams; Torsion; Influence of shear; Open sections and closed-open sections; Analytics; FEM*

O pomacima trupa kontejnerskih brodova opterećenih na uvijanje

Sažetak

Razmatran je analitički pristup uvijanju štapova otvorenog i otvoreno-zatvorenog tankostjenog presjeka s jednom ravninom simetrije. Primjenjena je teorija uvijanja štapova otvorenog tankostjenog presjeka s utjecajem smicanja, na temeljima klasične teorije Vlasova za štapove otvorenog tankostjenog presjeka, te Umanskog za štapove zatvoreno-otvorenog presjeka. Opće poprečno opterećenje djeluje u stijenama štapa, reducirano na momente uvijanja u odnosu na glavni pol (središte uvijanja/središte smicanja). Štap će biti opterećen na uvijanje s utjecajem smicanja u odnosu na glavni pol, te dodatno na savijanje zbog smicanja u horizontalnoj ravnini kroz glavni pol. Dobiveni analitički izrazi upotrebljeni su u analizi pomaka trupa modernog kontejnerskog broda opterećenog na uvijanje, kao i u parametarskoj analizi jednostavnih U profila. Dana je usporedba s metodom konačnih elemenata, s ljuskastim elementima.

Ključne riječi: *Teorija tankostjenih štapova; uvijanje; utjecaj smicanja; otvoreni i zatvoreno- otvoreni presjeci; analitika; MKE*

1. Introduction

In classical theories of torsion of thin-walled beams with open cross-sections warping of the cross-section due to shear is neglected [1,2,3].

Analogous to the advanced theories of bending, by an engineering approach [4,5,6,7,8], the concept of shear factors is expanded to the analyses of torsion [9,10,11,12,13,14].

In this paper, analytical expressions for displacements of thin-walled beams of open cross-sections subjected to torsion with influence of shear are applied in parametric analyses of simple U-sections [13,14]. In the case of closed-open sections [14,15,16], the analysis is given for modern container ship hull girder cross-sections [16,17,18]. Comparisons with the finite element method are provided.

2. Strains and displacements

The displacement of an arbitrary point $S(x,s)$ in the case of torsion of thin-walled beams of open sections with one axis of symmetry can be expressed as

$$u_S = -\frac{\mathrm{d}\alpha}{\mathrm{d}x}\omega - \frac{\mathrm{d}v}{\mathrm{d}x}y + \int_0^s \gamma_{x\xi}\,\mathrm{d}s\,, \tag{1}$$

where α is the angle of torsion, i.e. the rotation of the cross-section middle line as a rigid line with respect to a cross-section pole P; $\omega = \omega(s)$ is the sectorial coordinate with respect to the pole P; $v = v(x)$ is the displacement of the pole P in the y-direction, $y = y(s)$ is orthogonal coordinate; $\gamma_{x\xi} = \gamma_{x\xi}(x,s)$ is the shear strain in the middle surface; s is the curvilinear coordinate of the middle line; ξ is the tangential axis on the curvilinear coordinate s; Oxyz is the orthogonal coordinate system, where the z-axis is the axis of symmetry; $\varphi = \varphi(s)$ is the angle between the tangent ξ and the y-axis (Fig1);

$$\omega = \int_0^s h_P\,\mathrm{d}s\,, \qquad \mathrm{d}\omega = h_P\mathrm{d}s\,, \tag{2}$$

where $\omega = \omega(s)$ is the sectorial coordinate for the pole P; $h_P = h_P(s)$ is the distance of the tangent through the arbitrary point S at middle line from the pole P; $\omega(s=0) = 0$.

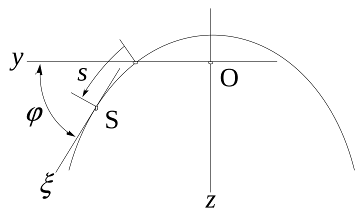

Fig.1 Cross-section coordinates
Slika 1. Koordinate poprečnog presjeka

Eq. (1) may be written as

$$u_S = \vartheta\omega - \gamma y + \int_0^s \gamma_{x\xi}\,\mathrm{d}s\,, \tag{3}$$

where

$$\vartheta = -\frac{\mathrm{d}\alpha}{\mathrm{d}x}, \qquad \gamma = \frac{\mathrm{d}v}{\mathrm{d}x}; \tag{4}$$

$\vartheta = \vartheta(x)$ is the relative angular displacement of the middle line as the rigid line with respect to the pole P and $\gamma = \gamma(x)$ is angular displacement of the middle line as the rigid line with respect to the z-axis.

Thus, it is assumed that the middle line displaces in the longitudinal direction due to warping, as in the case of ordinary theory of torsion, by the first member of Eq. (3), and in addition due to shear, by the second and third members of Eq. (3).

The displacements may be separated as

$$\alpha = \alpha_t + \alpha_s, \qquad v = v_s, \tag{5}$$

where $\alpha_t = \alpha_t(x)$ is the angular displacement of the cross-sections as plane sections with respect to the pole *P*, as in the case of classical theories of thin-walled beams of open cross-sections, whereas $\alpha_s = \alpha_s(x)$ and $v_s = v_s(x)$ are the additional displacements due to shear.

Then

$$\vartheta = \vartheta_t + \vartheta_s, \qquad \gamma = \gamma_s, \tag{6}$$

where

$$\vartheta_t = -\frac{\mathrm{d}\alpha_t}{\mathrm{d}x}, \qquad \vartheta_s = -\frac{\mathrm{d}\alpha_s}{\mathrm{d}x}, \qquad \gamma = \gamma_s = \frac{\mathrm{d}v_s}{\mathrm{d}x}. \tag{7}$$

The strain in the beam longitudinal direction may then be expressed as

$$\varepsilon_x = \frac{\partial u}{\partial x} = -\frac{\mathrm{d}^2\alpha}{\mathrm{d}x^2}\omega - \frac{\mathrm{d}^2 v}{\mathrm{d}x^2} y + \int_0^s \frac{\partial \gamma_{x\xi}}{\partial x}\mathrm{d}s. \tag{8}$$

3. Stresses and displacement

By ignoring the normal stresses in the transverse direction, the Hooke's low may be simplified as

$$\sigma_x = E\varepsilon_x \qquad \tau_{x\xi} = G\gamma_{x\xi}, \tag{9}$$

where *E* is the modulus of elasticity and *G* is the shear modulus.

Thus,

$$\sigma_x = -E\frac{\mathrm{d}^2\alpha}{\mathrm{d}x^2}\omega - E\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} y + \frac{E}{G}\int_0^s \frac{\partial \tau_{x\xi}}{\partial x}\mathrm{d}s. \tag{10}$$

From the equilibrium of a differential portion of the beam wall, if

$$\frac{\partial \tau_{x\xi}}{\partial x} = \text{const.}, \tag{11}$$

the following solution for the shear stress can be written

$$\tau_{x\xi} = \frac{E}{t}\left[\frac{\mathrm{d}^3 v}{\mathrm{d}x^3} S_z(s) + \frac{\mathrm{d}^3\alpha_u}{\mathrm{d}x^3} S_\omega(s)\right], \tag{12}$$

where

$$S_z(s) = \int_0^s y\,\mathrm{d}A, \quad S_\omega(s) = \int_0^s \omega\,\mathrm{d}A, \qquad \mathrm{d}A = t\mathrm{d}s; \tag{13}$$

$t = t(s)$ is the wall thickness.

Eq. (12) may be rewritten as

$$\tau_{x\xi} = -\frac{E}{t}\left(\frac{\mathrm{d}^3 v}{\mathrm{d}x^3} S_z^* + \frac{\mathrm{d}^3\alpha_u}{\mathrm{d}x^3} S_\omega^*\right), \tag{14}$$

where

$$S_y^* = \int_{s^*} z\,\mathrm{d}A^*, \qquad S_\omega^* = \int_{s^*} \omega\,\mathrm{d}A^*, \qquad \mathrm{d}A^* = t\mathrm{d}s^*, \qquad \mathrm{d}s^* = -\mathrm{d}s;$$

s^* is the curvilinear coordinate of the cut-off portion of the beam wall, the free edge, where $\tau_{x\xi} = 0$.

It is assumed that the normal stress given by Eq. (10) and shear stresses given by Eqs. (12) and (14) are constant across the wall thickness. According to the assumption that cross-sections maintain their shape during deformations, the St. Venant pure torsion may be included by a linearly distributed component $\tau_{x\xi,V}^{\mathrm{V}} = \tau_{x\xi}^{\mathrm{V}}(x,s)$,

$$\tau_{x\xi}^{\mathrm{V}} = \frac{M_t}{I_t}\eta, \qquad M_t = GI_t\frac{\mathrm{d}\alpha_t}{\mathrm{d}x} = -GI_t\vartheta_t, \tag{15}$$

where $M_t = M_t(x)$ is the moment of pure torsion;

$$I_t = \frac{1}{3}\int_L t^3\mathrm{d}s \tag{16}$$

Thus, the total shear stress $\tau_{x\xi}^{\mathrm{tot}} = \tau_{x\xi}^{\mathrm{tot}}(x,s)$ is

$$\tau_{x\xi}^{\mathrm{tot}} = \tau_{x\xi} + \tau_{x\xi}^{\mathrm{V}}. \tag{17}$$

4. Equilibrium equations

From the equilibrium of a finite portion of the beam wall the following equations can be written

$$\sum F_y = \int_L \frac{\partial\left(\tau_{x\xi}t\right)}{\partial x}\cos\varphi\,\mathrm{d}x\,\mathrm{d}s = 0, \ \sum M_P = \int_L \frac{\partial\left(\tau_{x\xi}t\right)}{\partial x}\mathrm{d}x\,h_P\mathrm{d}s + \frac{\mathrm{d}M_t}{\mathrm{d}x}\mathrm{d}x + m_P\mathrm{d}x = 0, \tag{18}$$

where

$$\int_L\left(\frac{\partial\tilde{M}_t}{\partial x}\right)\mathrm{d}s = \frac{\partial}{\partial x}\int_L \tilde{M}_t\,\mathrm{d}s\,\mathrm{d}x = \frac{\mathrm{d}M_t}{\mathrm{d}x}\mathrm{d}x,$$

$m_P = m_P(x)$ is the moment of torsion per unit length.

After integrating by parts, by substituting Eq. (12), Eqs. (18) become

$$EI_z\frac{\mathrm{d}^4v}{\mathrm{d}x^4} + EI_{z\omega}\frac{\mathrm{d}^4\alpha}{\mathrm{d}x^4} = 0, \qquad EI_{\omega z}\frac{\mathrm{d}^4v}{\mathrm{d}x^4} + EI_{\omega}\frac{\mathrm{d}^4\alpha}{\mathrm{d}x^4} = m_\omega, \tag{19}$$

where

$$I_z = \int_A y^2\mathrm{d}A, \qquad I_{z\omega} = I_{\omega z} = \int_A y\omega\,\mathrm{d}A, \qquad I_\omega = \int_A \omega^2\mathrm{d}A, \tag{20}$$

where

$$m_\omega = m_P + \frac{\mathrm{d}M_t}{\mathrm{d}x} = m_P + GI_t\frac{\mathrm{d}^2\alpha_t}{\mathrm{d}x^2} = m_P - GI_t\frac{\mathrm{d}\vartheta_t}{\mathrm{d}x}, \tag{21}$$

For the principal coordinate y and ω, when

$$I_{z\omega} = I_{\omega z} = 0, \tag{22}$$

Eqs. (19) become

$$\frac{\mathrm{d}^4v}{\mathrm{d}x^4} = 0, \qquad EI_\omega\frac{\mathrm{d}^4\alpha}{\mathrm{d}x^4} = m_\omega. \tag{23}$$

5. Internal forces and shear stresses

Integration of the shear stress components $\tau_{x\xi}$ over the cross-sections gives

$$\int_A \tau_{x\xi}\cos\varphi\,\mathrm{d}A = 0, \qquad M_\omega = \int_A \tau_{x\xi}h_P\,\mathrm{d}A, \tag{24}$$

where $M_\omega = M_\omega(x)$ is the sectorial moment of torsion with respect to the pole P.

Substitution of Eq. (14) into Eqs. (24) gives

$$\frac{\mathrm{d}^3 v}{\mathrm{d}x^3} = 0\,, \qquad M_\omega = -EI_\omega \frac{\mathrm{d}^3\alpha}{\mathrm{d}x^3}\,. \tag{25}$$

Referring to Eqs. (23) and (25) it can be written

$$\frac{\mathrm{d}M_\omega}{\mathrm{d}x} = -m_\omega\,. \tag{26}$$

Thus, by substituting Eq.(25) into (14), the shear stresses component $\tau_{x\xi}$ can finally be written as

$$\tau_{x\xi} = \frac{M_\omega S_\omega^*}{I_\omega t}\,. \tag{27}$$

6. Internal forces and normal stresses

Integration of the normal stresses over the cross-sections gives

$$B = \int_A \sigma_x \omega \mathrm{d}A\,, \qquad \int_A \sigma_x y\,\mathrm{d}A = 0\,, \tag{28}$$

where B is the bimoment. By substituting Eq.(10), the following equation by partial integrating, can be written as

$$B = -EI_\omega \frac{\mathrm{d}^2\alpha}{\mathrm{d}x^2} - B^\omega\,, \qquad EI_z \frac{\mathrm{d}^2 v}{\mathrm{d}x^2} - M_z^\omega = 0\,, \tag{29}$$

where

$$M_z^\omega = -m_\omega \frac{E}{GI_\omega}\int_L \frac{S_z^* S_\omega^*}{t}\mathrm{d}s\,, \qquad B^\omega = m_\omega \frac{E}{GI_\omega}\int_A \left(\frac{S_\omega^*}{t}\right)^2 \mathrm{d}A\,. \tag{30}$$

Referring to Eqs. (11),(25), (27), (29) and (09), the following equations may then be written

$$-EI_\omega \frac{\mathrm{d}^3\alpha}{\mathrm{d}x^3} = \frac{\mathrm{d}B}{\mathrm{d}x} + \frac{\mathrm{d}B^\omega}{\mathrm{d}x} = M_\omega\,, \qquad EI_z \frac{\mathrm{d}^3 v}{\mathrm{d}x^3} = \frac{\mathrm{d}M_z^\omega}{\mathrm{d}x} = 0 \tag{31}$$

and, according to Eq.(23),

$$-EI_\omega \frac{\mathrm{d}^4\alpha}{\mathrm{d}x^4} = \frac{\mathrm{d}^2 B}{\mathrm{d}x^2} + \frac{\mathrm{d}^2 B^\omega}{\mathrm{d}x^2} = \frac{\mathrm{d}M_\omega}{\mathrm{d}x} = -m_\omega\,, \qquad \frac{\mathrm{d}^4 v}{\mathrm{d}x^4} = 0\,. \tag{32}$$

Then, according to Eqs. (17) and (21), it may be written

$$M_P = M_\omega + M_t\,, \qquad m_P = -\frac{\mathrm{d}M_P}{\mathrm{d}x}\,. \tag{33}$$

Eqs. (29) may be expressed as

$$\frac{\mathrm{d}^2\alpha}{\mathrm{d}x^2} = -\frac{B}{EI_\omega} - \frac{\kappa_{\omega\omega}}{GI_P} m_\omega\,, \qquad \frac{\mathrm{d}^2 v}{\mathrm{d}x^2} = -\frac{\kappa_{y\omega}}{GW_P} m_\omega\,, \tag{34}$$

where

$$\kappa_{\omega\omega} = \frac{I_P}{I_\omega^2}\int_A \left(\frac{S_\omega^*}{t}\right)^2 \mathrm{d}A\,, \qquad \kappa_{y\omega} = \frac{W_P}{I_z I_\omega}\int_A \frac{S_z^* S_\omega^*}{t^2}\mathrm{d}A \tag{35}$$

are the shear factors with respect to the α-displacements and to the v-displacements during α-displacements, respectively;

$$I_P = \int_A h_P^2\,\mathrm{d}A\,, \qquad W_P = \frac{I_P}{h_0}\,; \tag{36}$$

are the polar second moment of area and the polar modulus of area, respectively; $h_P = h_P(s)$

is the distance between the tangent through the arbitrary point S at middle line from the principal pole P and h_0 is the distance of the tangent through the arbitrary starting point M_0 (where the principal coordinate ω is equal to zero) from the principal pole P.

7. Differential equations with separated displacements

According to Eqs. (5), Eqs. (34) can be separated as follows

$$\frac{\mathrm{d}^2\alpha_t}{\mathrm{d}x^2} = -\frac{B}{EI_\omega}, \qquad \frac{\mathrm{d}^2\alpha_s}{\mathrm{d}x^2} = -\frac{\kappa_{\omega\omega}}{GI_P} m_\omega . \tag{37}$$

According to Eqs. (32) and (7), one has

$$\frac{\mathrm{d}\alpha_s}{\mathrm{d}x} = -\vartheta_s = -\frac{M_\omega \kappa_{\omega\omega}}{GI_P}, \qquad \frac{\mathrm{d}v_s}{\mathrm{d}x} = \gamma_s = \frac{\kappa_{y\omega}}{GW_P} M_\omega . \tag{38}$$

It is assumed that the angular displacements θ_s and γ_s do not depend on the boundary conditions.

The first equation of Eqs. (37) is the well known equation of the classical theory of torsion of thin-walled beams; The second of Eqs. (37) and Eqs. (38) take into account the displacement due to shear.

Integrating Eqs. (38) gives

$$\alpha_s = \frac{\kappa_{\omega\omega}}{GI_P} B + C_\alpha, \qquad v_s = \frac{\kappa_{y\omega}}{GW_P} B + C_v , \tag{39}$$

where C_α and C_v are the integration constants.

Eqs. (39) can also be written as

$$\alpha_s = \frac{B}{GI_{Ps}} + C_\alpha , \qquad v_s = \frac{B}{GW_{Py}} + C_v , \tag{40}$$

where

$$I_{Ps} = \frac{I_P}{\kappa_{\omega\omega}}, \qquad W_{Py} = \frac{W_P}{\kappa_{y\omega}} \tag{41}$$

are the shear polar second moment of area and the shear polar modulus of area, respectively.

Boundary conditions with respect to the shear can be defined as follows, for the starting section A,

$$\alpha_s = 0 , \qquad v_s = 0 . \tag{42}$$

Hence, referring to (40),

$$C_\alpha = -\frac{B_A}{GI_{Ps}}, \qquad C_v = -\frac{B_A}{GW_{Py}} , \tag{43}$$

where B_A is the bimoment at $x = x_A$.

The total displacements then are

$$\alpha = \alpha_t + \frac{B - B_A}{GI_{Ps}}, \qquad v = \frac{B - B_A}{GW_{Py}} . \tag{45}$$

For the hinged sections it may be written

$$\alpha\big|_{x=x_A} = \alpha_t\big|_{x=x_A} = 0 \qquad \left(\alpha_A = \alpha_A^t = 0\right), \qquad \left.\frac{\mathrm{d}^2\alpha_t}{\mathrm{d}x^2}\right|_{x=x_A} = 0 \qquad \left(B_A = 0\right);$$

$$\alpha\big|_{x=x_B} = \alpha_t\big|_{x=x_B} = 0 \qquad \left(\alpha\big|_{x=x_B} = \alpha_B^t = 0\right), \qquad \left.\frac{\mathrm{d}^2\alpha_t}{\mathrm{d}x^2}\right|_{x=x_B} = 0 \qquad \left(B_B = 0\right). \tag{46}$$

For the free sections:

$$\left.\frac{\mathrm{d}^2\alpha_t}{\mathrm{d}x^2}\right|_{x=x_A}=0 \quad \left(B_A=0\right), \quad \left.\frac{\mathrm{d}^3\alpha_t}{\mathrm{d}x^3}\right|_{x=x_A}=0 \quad \left(M_{A\omega}=0\right). \tag{47}$$

8. Illustrative examples

The range of examples has been carried out by FEM using Autodesk Algor Simulation Pro in order to compare the results with those obtained analytically. Shell elements with 5 DOF are used. Mesh was generated with rectangle elements of $h/20$ width (in longitudinal direction) and $h/40$ height (in transverse direction). Transverse diaphragms of $t/10$ thickness are modeled with membrane elements every $h/2$ to prevent the distortion of the cross-section.

Due to symmetry, only one half of the beam is modeled (Fig. 2).

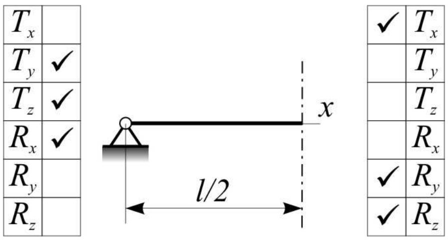

Fig. 2 Boundary conditions

Slika 2. Rubni uvjeti

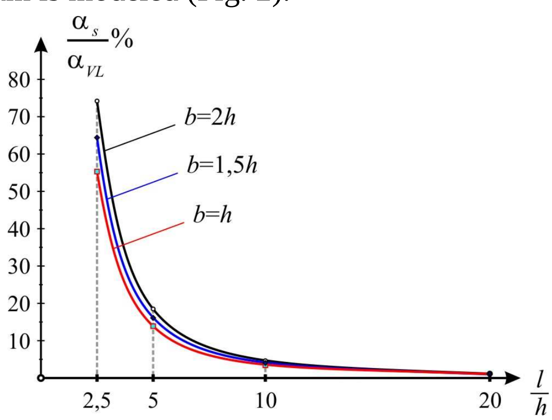

Fig. 3 Ratio of the angle of torsion due to shear and the angle of torsion by classical theory of torsion

Slika 3. Omjer kuta uvijanja zbog smicanja i kuta uvijanja po klasičnoj teoriji uvijanja

Relatively short beams, with $l/h \leq 10$ where considered.

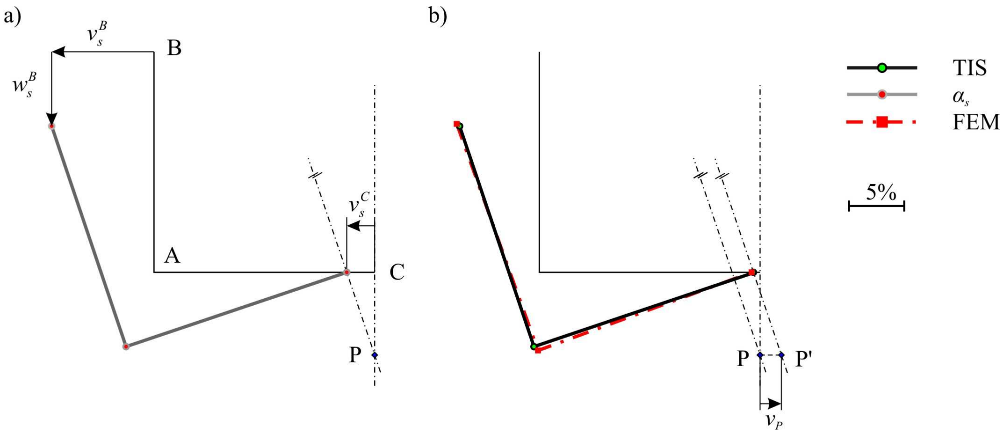

Fig. 4 Displacements of the cross-section middle line: a) due to angle of torsion due to shear, b) due to angle of torsion due to shear and horizontal displacement due to shear

Slika 4. Pomaci srednje linije poprečnog presjeka: a) zbog kuta uvijanja zbog smicanja, b) zbog kuta uvijanja zbog smicanja i horizontalnog pomaka zbog smicanja

Fig. 4 shows the displacements of the cross-section middle line due to shear for U-section beam with $l=5h$, $b=2h$ and $t_1=t_0=h/40$, for the beam loaded by moments of torsion per unit length, m_P, in comparison with FEM.

As shown, analytical results are in well agreement with those obtained by FEM.

Similar results are obtained for U-section beam with l=5h, b=1,5h and b=h. The angle of torsion due to shear achieves values of 18,5% of α_{VL} for b=2h, 16,1% of α_{VL} for b=1,5h, and 13,8% of α_{VL} for b=h, where α_{VL} is the angle of torsion obtained by classical theory [1].

As an example of a beam with open-closed cross-section the container ship with the following characteristics is considered [19]:

L=153 m; H=16,2 m; B=23,6 m; E=210000 MPa; G=80769 MPa; ν=0,3.

The properties of the cross-section are determined by the program SektorST [14]:

$$A = 2,318\cdot 10^6 \text{ mm}^2; \quad h_{Pd} = 6,033\cdot 10^3 \text{ mm} \; ; \quad I_t = 5,947\cdot 10^{12} \text{ mm}^4$$

$$I_\omega = 8,822\cdot 10^{21} \text{ mm}^6; \quad I_{Ps} = 1,221\cdot 10^{14} \text{ mm}^4; \quad W_{Py} = -1,882\cdot 10^{10} \text{ mm}^3$$

Fig. 5 shows the displacements of the midship cross-section due to shear. Ship is loaded with uniformly distributed moments of torsion per unit length, $m_P = 9,44\cdot 10^5$ N .

Maximal horizontal displacement due to shear obtained by presented theory is 5,4% of $v_{VL,\max}$ and the same displacement obtained by FEM is 3,5% of $v_{VL,\max}$, where $v_{VL,\max}$ is maximal horizontal displacement obtained by the classical theory [1].

Because of the additional displacement of the principal pole P, v_P, due to bending in the horizontal plane due to shear, the displacement due to shear of a point on the centerline, very near one half of the bottom height becomes 0 (Fig. 5b),

$$\alpha_s \left(h_{Pd} + h_d / 2\right) + v_P \approx 0$$

as it is noticed in [16].

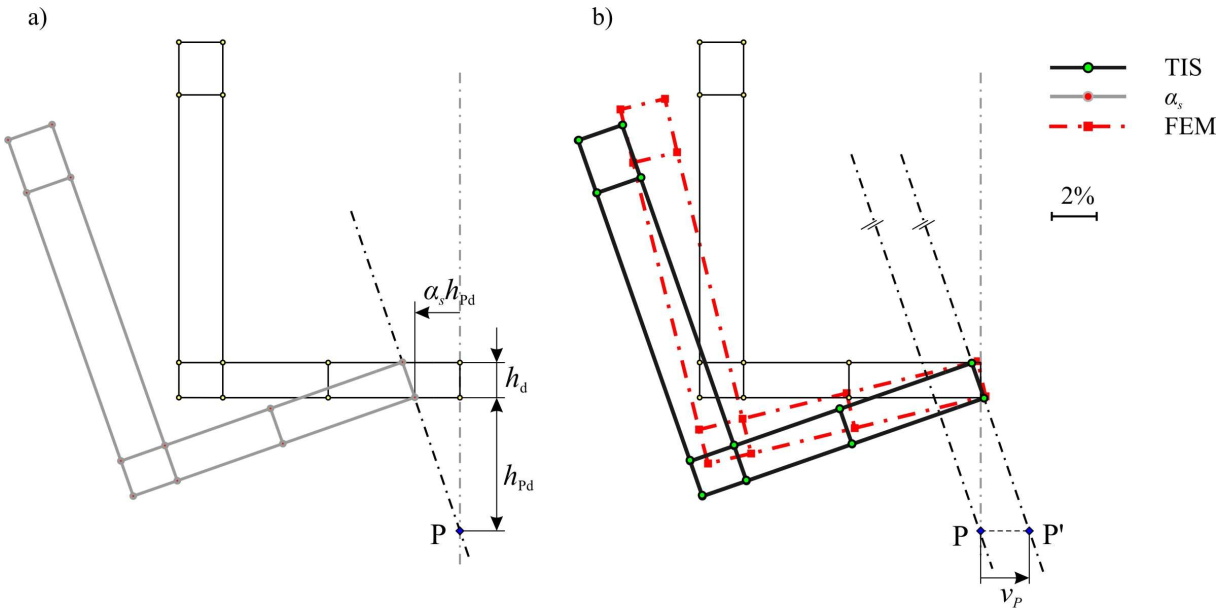

Fig. 5 Displacements of the midship cross-section: a) due to angle of torsion due to shear, b) due to angle of torsion due to shear and horizontal displacement due to shear

Slika 5. Pomaci srednje linije poprečnog presjeka broda: a) zbog kuta uvijanja zbog smicanja, b) zbog kuta uvijanja zbog smicanja i horizontalnog pomaka zbog smicanja

9. Conclusion

The theory of torsion of thin-walled beams with influence of shear for open sections with one axis of symmetry is applied in the analysis of displacements of beams with simple U-sections as well as container ship girder structures. Shear factors with respect to the torsion are given in the analytical parametric form in the case of simple U-sections, while for the real ship structures they are obtained numerically, by using the appropriate computer program.

It is shown that the beams with the single symmetrical section, such as U-section or closed open sections, as container ship sections, loaded to torsion by couples in the cross-section planes are also subjected to bending in the plane orthogonal to plane of symmetry.

For modern container ship structures, the shear influence on displacements is small, for real load conditions. For container ships with single side structures, this influence could be significant.

Comparisons of the numerical results with the finite element method have shown an acceptable agreement for engineering purposes, for early design stage processes.

Appendix

Properties of the U-section

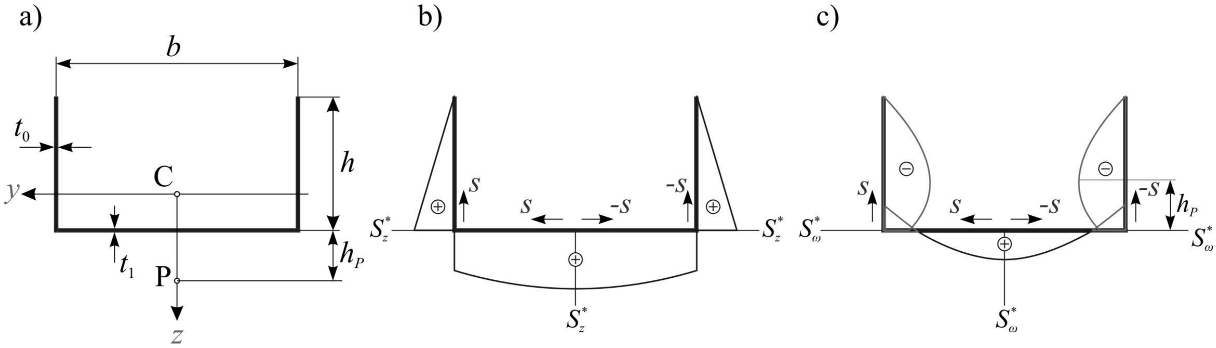

Fig. 6. U-section properties: a) middle line, b) distribution of S_z^* function, c) distribution of S_ω^* function

Slika 6. Značajke U-presjeka: a) srednja linija, b) raspodjela funkcije S_z^*, c) raspodjela funkcije S_ω^*

Distribution of the statical moments of the cut-off portion of area (Fig.6):

$$0 \le s \le h: \quad S_z^* = \frac{bt_0}{2}(h-s); \quad S_\omega^* = -\frac{b}{4}\left[2(h-h_P)-(h-s)\right](h-s)t_0;$$

$$0 \le s_y \le \frac{b}{2}: \; S_z^* = \frac{hbt_0}{2} + \frac{1}{2}\left(\frac{b^2}{4}-s^2\right)t_1; \quad S_\omega^* = -\frac{bt_0}{4}(h-2h_P)h + \frac{1}{2}\left(\frac{b^2}{4}-s^2\right)h_P t_1. \qquad (48)$$

Shear factors, according to Eqs. (35), are [13]:

$$\kappa_{\omega y} = -\frac{\left[18\psi + \rho^2(1+6\psi)^2\right]\cdot\left[10(5+6\psi)-2\rho^2\right]}{20\rho^2(2+3\psi)(1+6\psi)^2},$$

$$\kappa_{\omega\omega} = \frac{3\left[18\psi + \rho^2(1+6\psi)^2\right]\cdot\left[2\left(8+21\psi+18\psi^2\right)+3\psi\rho^2\right]}{10\rho^2(1+6\psi)^2(2+3\psi)^2}, \qquad (49)$$

where

$$A_1 = bt_1; \qquad A_0 = ht_0; \qquad \psi = \frac{A_0}{A_1}; \qquad \rho = \frac{b}{h}.$$

Closed-open cross-section

Sectorial coordinate for the pole P is

$$\omega = \int_s \left(h_P - \frac{q_i}{t}\right)\mathrm{d}s \qquad (50)$$

where q_i is shear flow in i-th closed contour obtained by solving the system of equation:

$$q_i \oint_i \frac{\mathrm{d}s}{t} - \sum_k q_k \int_{s_{ik}} \frac{\mathrm{d}s}{t} = A_i; \; i = 1,n \; , \qquad (51)$$

where

$$q_i = -\frac{T_i}{2G\vartheta}, \quad T_i = \tau_{x\xi} t ,$$

q_k is shear flow in the branch that belongs both to the *i*-th nad *k*-th closed contour and A_i is the area enclosed by *i*-th closed contour.

Then

$$I_\omega = \int_A \omega^2 \mathrm{d}A , \quad I_t = 4\sum_i q_i A_i , \quad S_z^* = S_{z0}^* - q_i^y , \quad S_\omega^* = S_{\omega 0}^* - q_i^\omega ,$$

where S_{z0}^* and S_{z0}^* are statical moments of the cross-section with fictitious cuts, and q_i^y and q_i^ω are unknown shear flows, obtained from the systems of Eqs. (52), respectively:

$$q_i^y \oint_i \frac{\mathrm{d}s}{t} - \sum_k q_k^y \int_{s_{ik}} \frac{\mathrm{d}s}{t} = \oint \frac{S_{z0_i}^*}{t} \mathrm{d}s; \qquad q_i^\omega \oint_i \frac{\mathrm{d}s}{t} - \sum_k q_k^\omega \int_{s_{ik}} \frac{\mathrm{d}s}{t} = \oint \frac{S_{\omega 0_i}^*}{t} \mathrm{d}s; \quad i = 1, n . \tag{52}$$

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